Inertial Navigation

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This paper will discuss the design and implementation of an inertial navigation system (INS) using an inertial measurement unit (IMU) and GPS. The INS is capable of providing continuous estimates of a vehicle's position and orientation. Typically IMU's are very expensive sensors, however this INS will use a "low cost" version costing only $5,000. Unfortunately with low cost also comes low performance and is the main reason for the inclusion of GPS into the system. Thus the IMU will use accelerometers and gyros to interpolate between the 1Hz GPS positions. All important equations regarding navigation are presented along with discussion. Results are presented to show the merit of the work and highlight various aspects of the INS.

I. Introduction

Navigation has been present for thousands of years in some form or another. The birds, the bees, and almost everything else in nature must be able to navigate from one point in space to another. For people, navigation had originally included using the sun and stars. Over the years we have been able to develop better and more accurate sensors to compensate for our limited range of senses. This paper will discuss work using one of these advanced sensors, an inertial measurement unit (IMU). This sensor, coupled with the proper mathematical background, is capable of detecting accelerations and angular velocities and then transforming those into the current position and orientation of the system.

Inertial Navigation Systems (INS) have been developed for a wide range of vehicles. Sukkarieh [1] developed a GPS/INS system for straddle carriers that load and unload cargo ships in harbors. When the carriers would move from ship to ship, they would periodically pass under obstructions that would obscure the GPS signal. Also, as the carriers got closer to the quay cranes, it became more difficult to get accurate positions due to the GPS signal being reflected about the cranes metal structure. This increases the time of flight of the GPS signal and results in jumps in the position. During these times the INS would then take over, and guide the slow moving carrier until a reliable GPS signal could be acquired.

Bennamoun et al [2] developed a GPS/INS/SONAR system for an autonomous submarine. The SONAR added another measurement to help with accuracy, and provided a positional reference when the GPS antenna got submerged and could not receive a signal.

Ohlmeyer et al [3] developed a GPS/INS system for a new smart munitions, the EX-171. Due to the high speed of the missile, update rates of 1 second from a GPS only solution were too slow, and could not provide the accuracy needed.

A. Outline

The first section of this paper will introduce inertial navigation. Then the IMU and GPS hardware will be covered. Finally experimental results using this INS will be presented.

II. Inertial Navigation

This section will cover strap-down inertial navigation by first describing the methods and equations. Next sources of error for these systems and how the kalman filter will be utilized to account for these errors.

A. Overview of Inertial Navigation Systems

A basic flow chart of how inertial navigation works is shown in Figure 1. However, this is not all that needs to be done to have an INS that works. There are many problems with noise and unbounded error that must be handled to get any meaningful result out of the INS.

Gimballed INS

The first type of INS developed was a gimballed system. The accelerometers are mounted on a motorized gimballed platform which was always kept aligned with the navigation frame. Pickups are located on the outer and inner gimbals which keep track of the attitude of the stabilized platform relative to the vehicle on which the INS is mounted. This setup has several detractors which make it undesirable.

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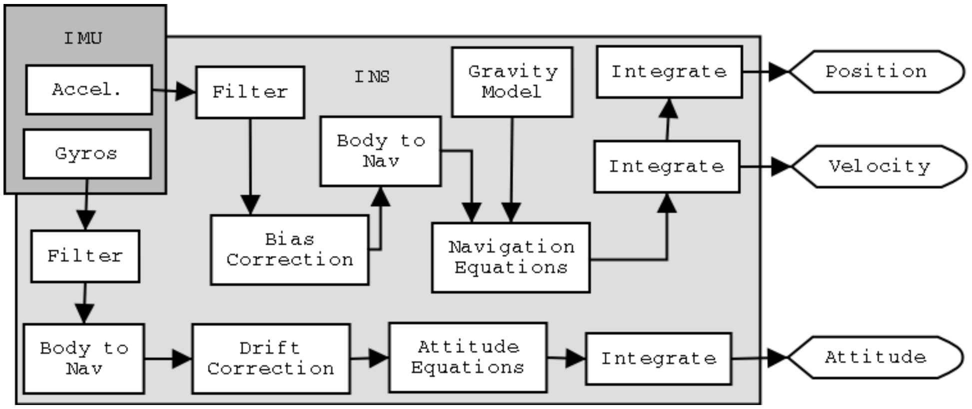

FIGURE 1 A flow chart of a strap-down INS which takes acceleration and rotation rates from the IMU and produces position, velocity, and attitude of the system.

- Bearings are not frictionless.
- Motors are not perfect (i.e. dead zones, etc.).
- Consumes power to keep the platform aligned with the navigational frame which is not always good on an embedded system.
- Cost is high due to the need for high quality motors, slip rings, bearings and other mechanical parts. Thus the typical customers for such systems were military uses on planes, ships, and intercontinental ballistic missiles.
- Recalibration is difficult, and requires regular maintenance by certified personnel which could be difficult on an autonomous vehicle. Plus any maintenance that must be performed on the system (i.e. replace bearings, motors, etc.) must be done in a clean room and then the system must go through a lengthy recertification process.

Strap-down INS

A strap-down system is a major hardware simplification of the old gimballed systems. The accelerometers and gyros are mounted in body coordinates and are not mechanically moved. Instead, a software solution is used to keep track of the orientation of the IMU (and vehicle) and rotate the measurements from the body frame to the navigational frame. This method overcomes the problems encountered with the gimballed system, and most importantly reduces the size, cost, power consumption, and complexity of the system.

B. Reference Frames and Rotations

Inertial navigation uses several reference frames, which are shown in Figures 2 and 3. To transition between the various reference frames, several rotation matrices are needed. The first one takes measurements in the body frame and puts them into the navigation frame,

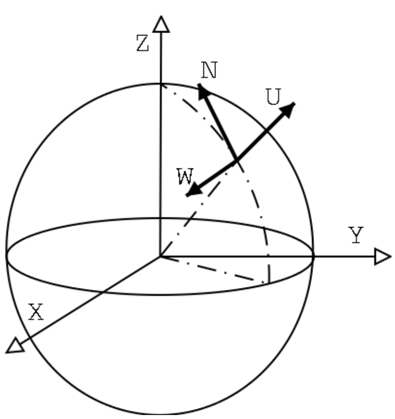

FIGURE 2 The XYZ frame is the inertial frame ECEF and the NWU frame is the local navigational frame, where the axes are north (N), west (W), and up (U).

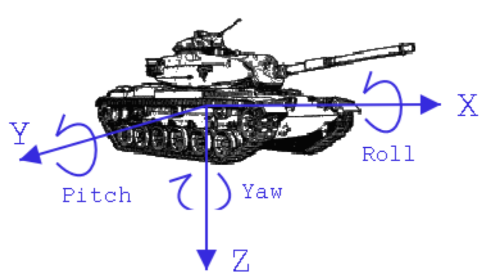

FIGURE 3 Body frame which is aligned with the axes of the IMU. The center of this frame is located at the origion of the navigational frame.

$$R_b^n = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & s\phi s\psi + c\phi s\theta c\psi \\ c\theta s\psi & c\phi c\psi + s\phi s\theta s\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (1)$$

where ϕ is roll, θ is pitch, and ψ is yaw. This rotation is the sequence 1-2-3, which is typically used in aerospace applications. This is a type 1 sequence which has singularities when the pitch is +/- 90 degrees since at this angle both the roll and yaw have similar effects. Thus for fighter aircraft which typically encounter this range, other methods must be included to account for this problem.

The next rotation will transform points from the ECEF frame to the navigation frame,

$$R_e^n = \begin{bmatrix} -s\phi c\lambda & -s\phi s\lambda & c\phi \\ -s\lambda & c\lambda & 0 \\ -c\phi c\lambda & -c\phi s\lambda & -s\phi \end{bmatrix} \quad (2)$$

where ϕ is latitude and λ is longitude. Now with these two rotations we can get another rotation, the one we really need.

$$R_b^e = R_n^e R_b^n \quad (3)$$

The last thing to remember with the above equation is that the inverse of any orthogonal rotation matrix is equal to its transpose. If a rotation matrix is not orthogonal (and this a problem with using Euler angles in navigation) then the previous statement is invalid.

C. Navigation Equations

Looking at Newton's second law of motion, a change in motion occurs as a force is applied to a body. Now, dividing both sides of the equation by the mass of the object results in the specific force.

$$\frac{f}{m} = a = S \quad (4)$$

In inertial navigation, accelerometers detect accelerations due to forces exerted on the body. These forces are typically referred to as specific forces (S). Thus reading from the IMU will be referred to as specific forces, which are independent of the mass. The navigation equations for the Earth Centered Earth Fixed (ECEF) system are shown below.

$$\begin{bmatrix} \dot{V}^e \\ \dot{P}^e \\ \dot{\Phi} \end{bmatrix} = \begin{bmatrix} -2\underline{\Omega}_{ie}^e & -\underline{\Omega}_{ie}^e\underline{\Omega}_{ie}^e & 0 \\ I & 0 & 0 \\ 0 & 0 & Q \end{bmatrix} \begin{bmatrix} V \\ P \\ \Phi \end{bmatrix} + \begin{bmatrix} R_b^e & R_b^e & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{SHC}^c \\ S^b \\ \omega \end{bmatrix} \quad (5)$$

$$\underline{\Omega}_{ie}^e = \begin{bmatrix} 0 & -\omega_{ie} & 0 \\ \omega_{ie} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (6)$$

where ω_{ie} is the rotation rate of the earth, R is a rotation matrix between different coordinate systems, P is the position and V is the velocity vector in the ECEF coordinate system as denoted by the superscript e. Also the attitude will be changed from euler's roll, pitch, and yaw to quaternions. Quaternions will help prevent the body to navigation rotation matrix, which transforms points from body frame to the navigational frame and back, from becoming non-orthogonal.

$$Q = \frac{1}{2}\begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \quad (7)$$

D. Sources of Error

This section will provide a quick overview of some difficulties present in inertial navigation. This will provide a better understanding for the difficulties encountered with the IMU.

Bias and Drift

These are the most devastating effectors on accuracy to an IMU. Drift rate for the gyros and accelerometer bias are small offsets which the IMU incorrectly reads, that must be properly accounted for. The bias has a quadratic effect on the position derived from the IMU.

$$error = \frac{1}{2} bias \cdot t^2 \quad (8)$$

TABLE 1. Positional error that results from biases after a time of 100 seconds and 30 mins.

Bias (m/s^2)	Error (m) t=100 sec.	Error (m) t = 30 mins
.1	500	162000
.01	5	16200
.001	.5	1620
.0001	.05	162

Looking at the Table 1 above it becomes apparent that determining the bias is of critical importance if any accurate measurement is expected.

The drift rate has a similar, and an equally massive impact on the position of a system. If a drift is not properly accounted for, and the IMU thinks it is rotating, then the

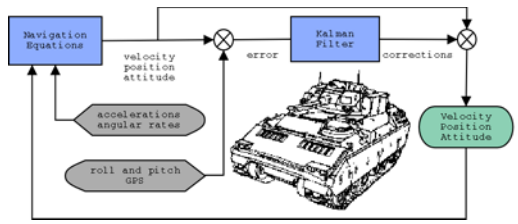

FIGURE 4 Overview of the extended kalman filter's integration with the INS.

navigation equations will not properly account for gravity and the system will think it is moving due to a maximum acceleration of 9.8 m/s^2 depending on how far the system has drifted.

Temperature

The IMU's accelerometers and gyros are sensitive to temperature as shown by Nebot and Durran-Whyte [4]. Thus as the temperature of the IMU changes, the associated bias and drift will change until the temperature reaches steady state or remains the same. This is not critical in our application, we just wait for the IMU to reach steady state before trusting the readings. However if this system was mounted in an aircraft which changed altitude and temperatures, this would be a problem.

Hysteresis

The drift rates and accelerometer biases tend to change each time the unit is switched on. This is due to the fact that measurements are noisy. Typically there is a low pass filter used to remove some of this noise before the measurements are used in the navigation equations (also realistically, there tends to be low pass filtering somewhere in the system due to hardware limitation because not everything has infinite bandwidth). When random noise is filtered, this produces what is called a random walk. The integration of this random walk will result in velocity and positions moving at different rates during different runs even though the IMU (and vehicle) are in the same orientation and experiencing the same accelerations during each run.

To give an idea of the performance of a strap-down system, the following quote is taken from an article [5] written by A. D. King, Chief Engineer of Navigation and Electro-optic Systems Division of Marconi Electronic Systems. Marconi produces INS for virtually all of the RAF's combat aircraft as well as many other systems.

> *"Many of these instrument errors vary each time you switch the system on - INS have good days and bad days. To characterize the performance of an INS, you have to resort to statistics, and take the r.m.s. total error from an ensemble of many representative missions. A typical standard expected from a 'good' INS produces an error that increases with time (not an entirely linear fashion), and reaches .6 miles after one hour (referred to as .6 nautical miles/hour system)."*

Vibrations

Vibration in a strap-down system can cause many problems with the INS. Generally great care must be taken to isolate the IMU from any resonance frequencies. In high precision systems, various tests must be done to try to identify what these frequencies are then design elaborate mounts to hold the IMU.

E. Extended Kalman Filter

In addition to the prefiltering of the IMU data, an extended kalman filter was developed to estimate the biases and drifts of the system and then update the navigational solution. The full kalman filter equations will not be presented here due to limited space, but an overview of the process is shown in Figure 4 and further information can be found in Brown and Hwang [6].

The error model was developed based on derivations by Chatfield [7] and Rogers [8]. This filter model is small compared to other authors have anywhere between 20 and 50 different states, depending on how their navigational models were defined. Note that there is also the inclusion of two sets of terms which now makes this an extended kalman filter model. The terms are the errors in bias on the accelerometers, and drift of the gyros. Each is modelled as a random walk (or could have modelled them as a markov process), where the terms with the subscript N on the far right of the equation are zero mean, random white noise with the appropriate standard deviation. The purpose of this is to estimate these new parameters, since they are difficult to determine, and (as in the case of the bias) change greatly depending on temperature, time, and orientation.

$$\begin{bmatrix} \delta\dot{V} \\ \delta\dot{P} \\ \delta\dot{\phi} \\ \delta\dot{S}^b \\ \delta\dot{\omega}^b \end{bmatrix} = A \begin{bmatrix} \delta V \\ \delta P \\ \delta\phi \\ \delta S^b \\ \delta\omega^b \end{bmatrix} + B \begin{bmatrix} \delta S_N^b \\ \delta\omega_N^b \end{bmatrix} \tag{9}$$

FIGURE 5 Sensors used in the INS. (left) The Crossbow DMU-HDX which is a solid state vertical gyro capable of measuring angular rates and accelerations on all three axes. It also has the capability of measuring the roll and pitch of the device too. (right) Garmin 16LVS OEM GPS which is both a reciever and antenna.

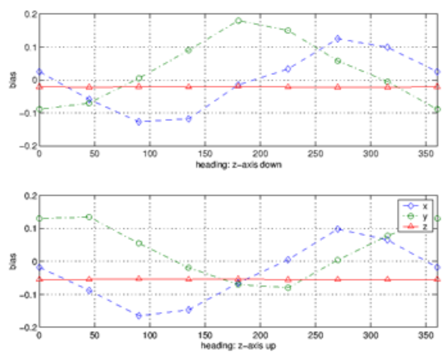

FIGURE 6 This is a plot of the biases as the IMU was rotated around the z-axis (yaw). Rotations around the other axes would also efect the biases, thus this mapping is ont useful since the values are changing nonlinearly.

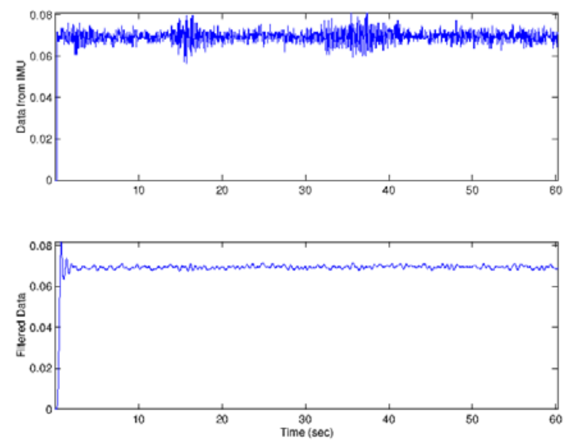

FIGURE 7 Comparison of the unfiltered data (top) produced by the IMU and the filtered data (bottom) using the Chebyshev II filter. Data is from one of the accelerometers while the IMU is sitting still on a table.

$$A = \begin{bmatrix} -2\underline{\Omega}^e_{ie} & \underline{\Omega}_{ie}\underline{\Omega}_{ie} & \underline{S}^e & R^e_b & 0_{3x3} \\ I_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & \underline{\Omega}^e_b & 0_{3x3} & -R^e_b \\ & & 0_{6x15} & & \end{bmatrix} \tag{10}$$

$$B = \begin{bmatrix} 0_{6x6} \\ I_{6x6} \end{bmatrix} \tag{11}$$

$$\underline{S}^e = \begin{bmatrix} 0 & -S^e_z & S^e_y \\ S^e_z & 0 & -S^e_x \\ -S^e_y & S^e_x & 0 \end{bmatrix} \tag{12}$$

III. Hardware

This section will provide an overview of the two primary sensors, the IMU and GPS shown in Figure 5.

A. Crossbow IMU

The IMU is a solid state vertical gyro (DMU_HDX) from Crossbow Technologies intended for airborne applications such as UAV control, Avionics, and Platform Stabilization. This high reliability, strap-down inertial subsystem provides attitude measurement with static and dynamic accuracy comparable to traditional spinning mass vertical gyros. Data will be transmitted by the DMU digitally via a serial connection (RS-232). The gyros on the Crossbow IMU are low cost, low performance MEMS (Mechanical Electrical Micro-Systems) gyros. These gyros are much less expensive to produce, but performed at least an order of magnitude worse than another low cost IMU system being developed by Dr. Crane here at U.F. That system uses an IMU developed from Honeywell which has ring laser gyros. Unfortunately, the gyro performance is a critical element in accounting for gravity in the system.

Prefiltering IMU Data

The data produced by the IMU is extremely noisy, thus a filter was designed. Matlab's signal toolbox was used to accomplish this task. The toolbox is capable of designing

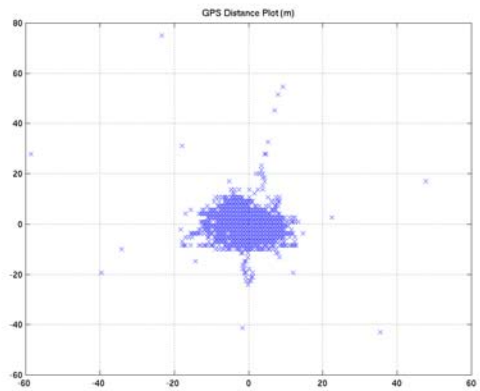

FIGURE 8 This is a test of the GPS accuracy. The GPS was set in a stationary location for 4 hours. The center of the plot was taken to be the average latitude and longitude reported by the sensor. Then the corresponding distances from the average were calculated. This GPS reciever is capable of providing the standard 10 meter accuracy 95% of the time.

all of the classic FIR and IIR filters. A IIR filter was decided on since it produces the same results as an FIR filter but with a much lower order. This lower order results in a less computational process. The following specifications for the filter were decided on: pass band value of 2Hz, stop band of 3 Hz, and a stop band attenuation of -50 dB.

Additionally, the desired filter should not have any ripple in the pass band range, thus the Equiripple, Elliptic, and Chebyshev I filters were eliminated as possible designs. The remaining Butterworth and Cheyshev II filters were looked at. After much testing with various options, the Chebyshev II filter was settled one as the best on for the job and its performance can be seen in Figure 7.

B. Garmin GPS

The GPS system used in this work is the Garmin 16LVS. Garmin is a common name in commercial civilian GPS systems, and this OEM device has performance that is on par with all other GPS systems available currently (i.e. accuracy of about 10 m 95% of the time) as shown in Figure 8. However this GPS was specifically bought because it included a WAAS (Wide Area Augmentation System) filter which should increase the accuracy to less than 3 m 95% of the time.

WAAS [9] utilizes ground stations which detect and send GPS error information to a Master Control site. The Master Control site uses this information to compute in order of importance or effect:

1.Integrity information
2.Ionospheric and Tropospheric delays
3.Short-term and long term satellite clock errors
4.Short-term satellite position error (Ephemeris)
5.Long term satellite position error (Almanac)

This information is relayed to two WAAS geosynchronous Inmarsat satellites (AOR-W and POR) from the Master Control Stations and is re-broadcast to user receivers as a grid of corrections. From this grid, a GPS receiver interpolates the proper Ionospheric correction based on its position in the grid. The "extrapolation" of this information outside the WAAS coverage is less and less precise -to the point of INDUCING errors. Other errors are not location dependant.

The WAAS correction information is different than RTCM corrections (transmitted by the Coast Guard for uses in DGPS) because WAAS decomposes the errors into their primary elements (Iono, clock, & ephemeris). RTCM, on the other hand, broadcasts pseudorange corrections which are the sum of all error sources as observed by the RTCM reference station. This information is only valid relatively close to the reference station. This is why spatial decorrelation is such a large factor for RTCM, but not for WAAS (thus the reason it is "wide area" augmentation).

IV. Results

The experiment is broken up into two parts. The first part is the navigation solution which does not utilize the kalman filter or the GPS positional corrections. The second part will include these so the limitation of the IMU and benefits of the kalman filter and GPS can be seen.

A. Test Setup

The experiments were conducted using a car with the IMU and GPS mount on it. A laptop was connected to both sensors and recorded the data. The data was then taken and analyzed in Matlab using the proceeding equations.

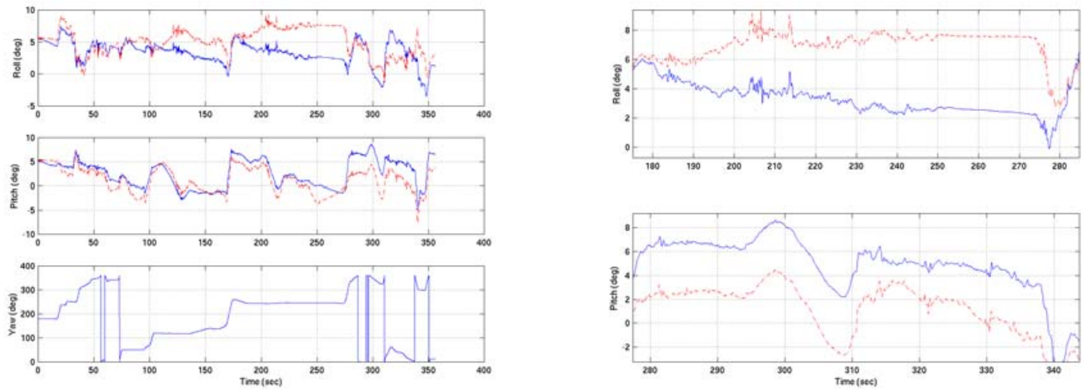

FIGURE 9 INS attitude solution with out extended kalman filter. The estimated roll, pitch, and yaw are shown by the solid line, while the true roll and pitch reported by the IMU is the dashed line.

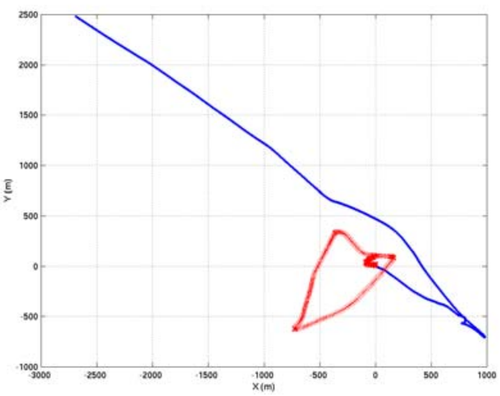

FIGURE 10 INS results without GPS and kalman filter integrated into the system.

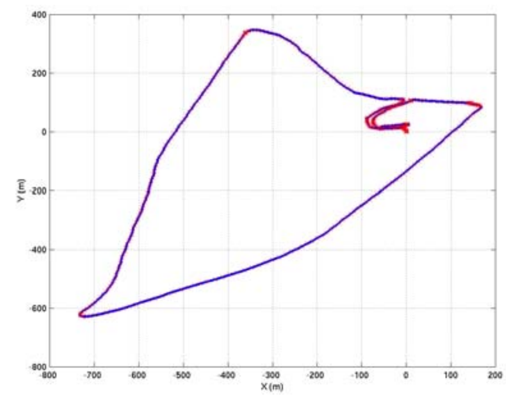

FIGURE 11 INS results with GPS and kalman filter integrated in to the system.

B. Navigational Solution Only

The first set of results was without the use of the extended kalman filter, to see if it was really necessary. The results of estimating the roll, pitch, and yaw without any corrections is shown in Figure 9. The estimated angles appear to track the true angles to an acceptable degree. The IMU is capable of reporting it's true roll and pitch, but not yaw. Assuming the performance between estimating the yaw angle and the pitch and roll angles are the same, it should not be necessary to require a compass to update the true yaw angle. The couple degrees of error should not effect INS results much since the car is traveling on flat roads.

The performance changes when we look at figure 10. The GPS and INS (i.e. using the navigation equations and IMU data only) differ greatly. Thus the GPS with the kalman filter must be included into the INS to give any good results.

C. GPS/INS

After the inclusion of the GPS and kalman filter, the plot shown in Figure 11 is much better. The GPS and INS lie right on top of each other. Taking a closer look at this plot, Figure 12 and Figure 13 show that the two do not really lie exactly on top, but rather the INS transitions smoothly through the GPS points.

Looking specifically at Figure 12, it can be seen that the IMU is picking up some of the accelerations in the turn and shifted the position left of the GPS points. But going into the turn and once the turn is completed, the INS and GPS positions merge back together.

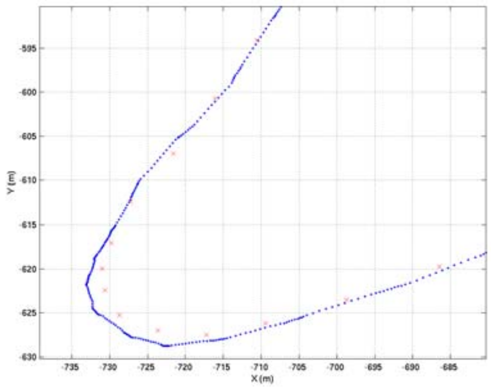

FIGURE 12 This plot shows the interpolating capabilities of the INS system in X and Y.

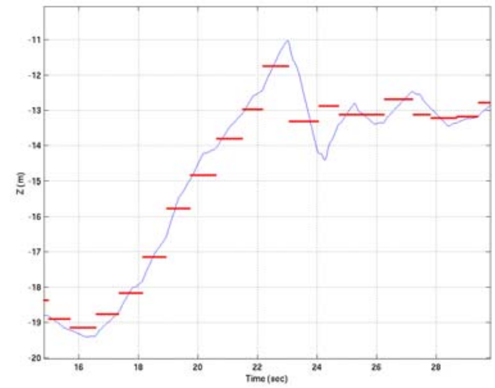

FIGURE 13 This plot shows the interpolating capabilities of the INS system in Z.

Figure 13 is better example showing how the INS is able to take the discrete GPS position and the accelerations from the IMU and fit a curve through the two. This level of continuos positioning can not be offered by GPS alone.

Finally the distances traveled during the experiment were calculated and the results were close as shown in Table 2. The car's odometer was felt to be the most accurate and the GPS and INS distances are on either side of the value.

TABLE 2. Distances traveled as reported by the different systems.

	GPS	INS	Car
km	4.89	5.16	
miles	3.04	3.21	3.1

The extended kalman filter attempts to estimate the biases and drifts present in the system to increase the accuracy of the system. However there appeared to be no difference between using the estimated biases and drifts estimated from the filter or using constant ones. This is attributed to the excessive amount of noise from the low cost IMU. Chatfield's [7] work was the prime motivator for including these terms in the extended kalman filter, but he assumed measurement that were much better (i.e. less noisy) than the ones being produced by the Crossbow IMU. Thus this part of the kalman filter could be eliminated to reduce computational expense with no loss of performance.

V. Conclusions

This paper has shown the effective combination of two different sensors (GPS and IMU) each with their own strengths and weaknesses. The "low cost" IMU used in this work is not capable of running by itself and providing any reasonable positioning information. GPS provides good results, but is only capable of determining position every second. The two sensors combined has the capability of producing good estimates of position in between the one second updates.

Acknowledgments

The research conducted in this paper could not have been accomplished with out the help of the Machine Intelligence Lab at University of Florida who donated the use of the IMU.

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