Development of a Sliding Mode Control System with Extended Kalman Filter Estimation for *Subjugator*[1]

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The University of Florida's autonomous submarine Subjugator is currently controlled by a series of proportional derivative controllers. This control scheme requires much maintenance in tuning and retuning gains whenever the smallest change to the submarine is made. This work describes a sliding mode controller designed to replace the current control system which is robust to these small changes, thus requiring less maintenance. Also this system will incorporate an extended Kalman filter which will estimate the submarine's position. The incorporation of these two new technologies give Subjugator several important capabilities that were previously absent. Simulations of the new control system are provided to highlight the advantages.

I. Introduction

Submarine navigation and control is a difficult and complex topic. There are many difficulties that need to be over come before the submarine will perform as desired. Some of the difficulties present in submarine control are due to disturbances and uncertainty. Internal disturbances arise from, for example, the interaction of the compass with the magnetic fields produced by the motors. External disturbances are produced by currents (or the jets in a pool) that interfere with the submarine. Complex hydrodynamic forces that are difficult to calculate make model based solutions to control and filter design difficult. This work is one such attempt to over come the difficulties and achieve control of the autonomous submarine *Subjugator*.

A. Subjugator

The University of Florida's Engineering Department has an autonomous submarine *Subjugator*, shown in Figure 1. The submarine is operated by a group of electrical and mechanical engineering student who enter the submarine in an annual competition sponsored by Autonomous Unmanned Vehicle Society International (AUVSI) and the Office of Naval Research (ONR).

The original control system for *Subjugator* was a series of linear PID controllers (one for each desired direction of movement). The control gains were tuned by hand through a trial and error method. The desired states being controlled were the motor's speed (not the submarine's speed), depth, and the heading. This was due to the fact that there were no sensors on board which could detect position or velocity. Thus the only real system states that were capable of being controlled were the depth and heading.

This method resulted in a control system that performed "in the ball park" of what was desired. However when any noise or disturbance would interfere with the system, all performance was lost. The designers had to "out think" the problem by programming cleaver hacks into the control software to handle sudden disturbances in the system to counter act this problem.

B. Prior Work

Many researchers have used various tools and methods to control submarines. Kiriazov et al [1], Dougherty and Woolweaver[2], Lam and Ura [3], and Kreuzerand Pinto [4] have used sliding mode for submarine control. Yuh and Nie [5] have designed an adaptive method loosely based on sliding mode control while Coute and Serrani [6] have used H infinity. Guo and Huang [7] used fuzzy logic and genetic algorithms. Appleby et al [8] compared several linear and nonlinear techniques of submarine controller design. All of these researchers have meet with varying degrees of success.

C. Outline

First an overview of submarine dynamics and disturbances are presented. Next the development of the sliding mode controller and extended Kalman filter are provided. Finally the results of a Matlab simulation are presented and discussed.

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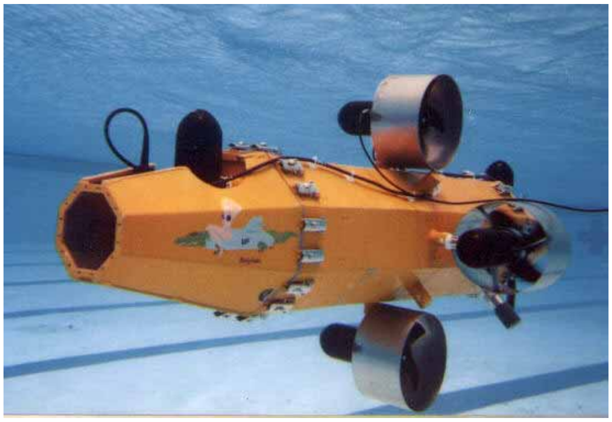

FIGURE 1. The submarine *Subjugator* running underwater.

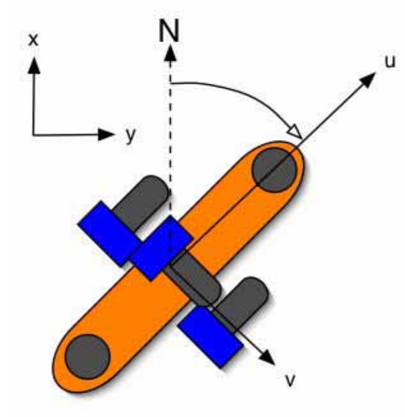

FIGURE 2. A free body diagram of *Subjugator*.

II. Submarine Dynamics

This section will cover one representation of the complex nonlinear dynamics associated with submarines. Many authors have looked at submarine dynamics and stability using various methods: Nahon [9], Brucher and Rydill [10], Popoulias et al [11], and Lenard and Marsden [12].

A. Simple Equations of Motion

The following model of a submarine is a simple one. The true differential equations of motion for a submarine are very complex, coupled, and highly nonlinear. However few authors uses these and prefer the simpler version since it makes controller and filter design much easier while still maintaining the important characteristics of submarine behavior.

$$m\dot{q} = \tau - F_d \tag{1}$$

$$X = \begin{bmatrix} q & x \end{bmatrix}^T \tag{2}$$

The state vector X is a composite of two vectors. The generalized coordinates q are measured in body coordinates and the x states are measured in world coordinates (see Figure 2). They have the following relationship:

$$\dot{x} = \begin{bmatrix} R_{123} & 0 \\ 0 & T \end{bmatrix} q \tag{3}$$

$$R_{123} = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & s\psi s\phi + c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi + s\phi s\theta s\psi & s\theta s\psi c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{4}$$

$$T = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & \frac{s\phi}{c\theta} & \frac{c\phi}{c\theta} \end{bmatrix} \tag{5}$$

$$q = \begin{bmatrix} u & v & w & p & q & r \end{bmatrix}^T \quad \text{and} \quad x = \begin{bmatrix} x & y & z & \phi & \theta & \psi \end{bmatrix}^T \tag{6}$$

where q is a vector of rate terms in body coordinates, x is vector of position terms in world coordinates, ϕ is roll, θ is pitch, and ψ is yaw.

B. Hydrodynamics

The hydrodynamic forces exerted on the submarine are given by Morrison's Equation

$$F_d = C_d \frac{1}{2} \rho_w D |q| q + C_m \rho_w A \dot{q} \tag{7}$$

where ρ_w is the density of the water, C_d is the dampening coefficient, D representative length (chord or foil), C_m is the hydodynamical inertia coefficient, A is the cross sectional area, and q and $\dot{q}$ are the relative velocities and accelerations between the submarine and the water.

C. Complete Equations of Motion

Since it is common to assume the water is stationary, u is in reference to the vehicle. Thus the second half of (7) can be moved into the mass matrix and is referred to as the added mass. The first half of the equation is the dampening coefficients which is contained in the hydrodynamic drag term (F_d) in equation (1). The equations of motion (1) can be rewritten so that the mass matrix now includes the acceleration terms from (7).

$$(m + C_{mi} \rho_w A_i) \dot{q}_i + C_{di} \frac{1}{2} \rho_w D_i |q_i| q_i = \tau_i \tag{8}$$

where the terms after the mass (m) are the added mass. This equation put into matrix form becomes:

$$M\dot{q} + C(q)q = \tau \tag{9}$$

$$M = diag(m_x, m_y, m_z, I_x, I_y, I_z) \tag{10}$$

$$C(q) = diag\left(C_d\frac{1}{2}\rho_w D_x|p|,\ C_d\frac{1}{2}\rho_w D_y|q|,\ C_d\frac{1}{2}\rho_w D_z|r|,\ \ldots\right) \quad (11)$$

The τ on the right side of (8) and (9) is a vector composed of forces and torques to the system. The kinematic relationship between these and *Subjugator*'s engines are

$$\tau = Bu \quad (12)$$

$$\tau = \left[F_x\ F_y\ F_z\ T_x\ T_y\ T_z\right]^T \quad (13)$$

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & \frac{w}{2} & -\frac{w}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{L}{2} & -\frac{L}{2} \\ \frac{w}{2} & -\frac{w}{2} & 0 & 0 & 0 & 0 \end{bmatrix} \quad (14)$$

$$u = \left[f_{xL}\ f_{xR}\ f_{yT}\ f_{yB}\ f_{zF}\ f_{zB}\right]^T \quad (15)$$

where w and L are the width and length of the submarine. The control forces in u are the left, right, top, bottom, front, and back motor respectively.

Problem, the C matrix values are near to impossible to determine experimentally and impossible to analytically calculate. These values vary with orientation, water flow direction, and speed. Also, due to the viscosity of water, the mass of the submarine changes with speed, and direction due to the water molecules "sticking" to the submarine. Since water is heavy, this "added mass" can change the overall mass of the submarine substantially. These problems will be dealt with through the use of sliding mode control and possibly an extended kalman filter could estimate the current values (although this was not done in this work).

III. Submarine Control and Estimation

This section will give a description of the sliding mode controller and Kalman filter designed for *Subjugator*.

A. Proportional Derivative

One of the simplest and most popular controllers implemented is the proportional derivative (PD) controller. The proportional gain is responsible for eliminating error in a system, while the derivative gain adds dampening.

$$u = k_p e + k_d \dot{e} \quad (16)$$

where the control effort is u, k_p is the proportional gain, k_d is the derivative gain, e is the error, and $\dot{e}$ is the error rate.

There are many different methods to arrive at the desired controller gains however these are typically model based. Thus the performance of the controller is proportional to the amount of error in the model. One of the largest sources of error is the hydrodynamic terms for the submarine. These are often impossible to calculate except for only simple structures.

Another method for gain determination (and the one used for *Subjugator*) is trial and error. However the gains are only valid for the environment in which they were determined and obviously nothing can be said about stability. Thus moving the submarine from a pool, where the gains were calculated, to an open body of water may result in instability due to surface wave and current interaction.

B. Sliding Mode

Nonlinear model based control systems offer a level of dynamic capabilities which linear techniques are incapable of providing when dealing with parameter uncertainties and unmodelled dynamics. Sliding mode [13], which has been studied in the Soviet Union for many years, is categorized as a variable structure control system which has excellent stability, robustness, and disturbance rejection characteristics. This type of control is not new to submarines, in fact it is widely used due to its capability to overcome modeling errors (due in this case to the hydrodynamic terms and modeling as an uncoupled system). Sliding mode has been used in: spacecraft [14], robotics [15], missiles [16], and many other applications where modelling error is a concern.

Sliding Mode for Subjugator

The sliding surface (s) defined by Slotine [17] is typically given by:

$$s = \dot{\tilde{x}} + \lambda\tilde{x} \quad (17)$$

where

$$\tilde{x} = x - x_{desired} \quad (18)$$

Here q is substituted for $\dot{x}$ and x is rotated from world coordinates to body coordinates. The new sliding surface is now:

$$s = \tilde{q} + \lambda\tilde{x}_B \quad (19)$$

The sliding surface in (19) gives the controller a PD type of action. This is used so that The is the error between the current state and the desired state defined by

The equivalent control effort ($\hat{\tau}$), is

$$\hat{\tau} = \hat{C}(\tilde{q})\tilde{q} - \lambda\hat{M}\tilde{q} \quad (20)$$

where $\hat{C}(\tilde{q})$ and $\hat{M}$ are estimates of the mass and dampening matrix. The equivalent control can be viewed as a feed forward type of control. Here the terms deal with the dampening forces and momentum of the system.

Now the final form of the sliding mode controller will take the form of:

$$\tau = \hat{\tau} - Ksat\left(\frac{s}{\phi}\right) \quad (21)$$

$$sat(a) = \begin{cases} a & |a| < 1 \\ \mathrm{sgn}(a) & otherwise \end{cases} \quad (22)$$

where $Ksat(s/\phi)$ is a vector of $k_i \mathrm{sgn}(s_i/\phi)$ gains and ϕ is a positive definite scalar term.

Stability Analysis

This section has two purposes, showing stability of the control system in the presence of modelling uncertainty and determining the control gain *K*. This is accomplished by choosing a positive definite Lyapunov based on the error dynamics of the system, then taking its derivative to shown that the equation is negative definite.

$$V = \frac{1}{2}s^T M s \quad (23)$$

$$\dot{V} = s^T M \dot{s} < 0 \quad (24)$$

$$s^T\left(M\dot{\tilde{q}} + \lambda M\tilde{q}\right) = s^T(\tau - C(\tilde{q})\tilde{q} + \lambda M\tilde{q}) \quad (25)$$

$$s^T(\hat{\tau} - K\mathrm{sgn}(s) + \lambda M\tilde{q} - C(\tilde{q})\tilde{q}) \quad (26)$$

$$s^T(\hat{C}(\tilde{q})\tilde{q} - \lambda\hat{M}\tilde{q} - K\mathrm{sgn}(s) + \lambda M\tilde{q} - C(\tilde{q})\tilde{q}) \quad (27)$$

where the $\hat{C}(\tilde{q})\tilde{q}$ and $\hat{M}$ are analytical estimates and $C(\tilde{q})\tilde{q}$ and M are the real dynamics of the system. Now if the estimates of the dampening and mass matrices were perfect, the dynamics would cancel out and the equation would only contain $-K\mathrm{sgn}(s)$. Since this equation would be negative definite, the derivation would be complete. However this does not happen. Thus the following substitutions are made:

$$\tilde{M} = \hat{M} - M \quad \text{and} \quad \tilde{C}(\tilde{q})\tilde{q} = \hat{C}(\tilde{q})\tilde{q} - C(\tilde{q})\tilde{q} \quad (28)$$

$$s^T(\tilde{C}(\tilde{q})\tilde{q} - \lambda\tilde{M}\tilde{q} - K\mathrm{sgn}(s)) < 0 \quad (29)$$

$$s^T(\tilde{C}(\tilde{q})\tilde{q} - \lambda\tilde{M}\tilde{q}) - \sum_1^n k_i|s_i| \quad (30)$$

Here two additional substitutions are made. The first is realizing the group of terms on the left half of the equation represent the disturbance forces from modelling error. Second, the typical equation for the gain *K* can be substituted into the equation.

$$F = \left|\tilde{C}(\tilde{q})\tilde{q} - \lambda\tilde{M}\tilde{q}\right| \quad \text{and} \quad K = F + \eta \quad (31)$$

$$s^T F - \sum_1^n (F_i + \eta_i)|s_i| < 0 \quad (32)$$

$$-\sum_1^n \eta_i|s_i| < 0 \quad (33)$$

Equation (33) is negative definite since η_i are strictly positive constants. The sliding condition guarantees that the surface $s = 0$ is reached in finite time and the system remains on the sliding surface. The chattering implied by the controller's use of $\mathrm{sgn}(s)$ can be eliminated by using a smooth function $sat(s)$.

C. Extended Kalman Filter

Subjugator has the capability to measure all states except velocity and position in the x and y direction. The velocity measurement could be obtained through the use of low cost flow meters (in relatively calm waters) or very expensive velocity Doppler. The Kalman filter presented here is not a solution by itself, but a small step forward. This implementation would still be effected by environmental disturbances (waves, currents, and pool jets).

For a nonlinear system were the system model and measurement model are:

$$\dot{x} = a(x, u, t) + G(t)w \quad (34)$$

$$z = h(x, t) + v \quad (35)$$

The continuos extended Kalman filter equations (which can be found in Brown et al [18] or Lewis [19]) for the estimate update, error covariance update and Kalman gains are:

$$\dot{\hat{x}} = a(\hat{x}, u, t) + K[z - h(\hat{x})] \quad (36)$$

$$\dot{P} = AP + PA^T + GQG^T - PH^T R^{-1} HP \quad (37)$$

$$K = PH^T R^{-1} \quad (38)$$

where the jacobians are:

$$A = \frac{\partial a(\hat{x}, u, t)}{\partial x} \quad \text{and} \quad H = \frac{\partial h(\hat{x}, t)}{\partial x} \quad (39)$$

Here the $a(\hat{x}, u, t)$ equations are the submarine's equations of motion and the jacobian of $h(\hat{x}, t)$ does not have to be taken since it is already linear.

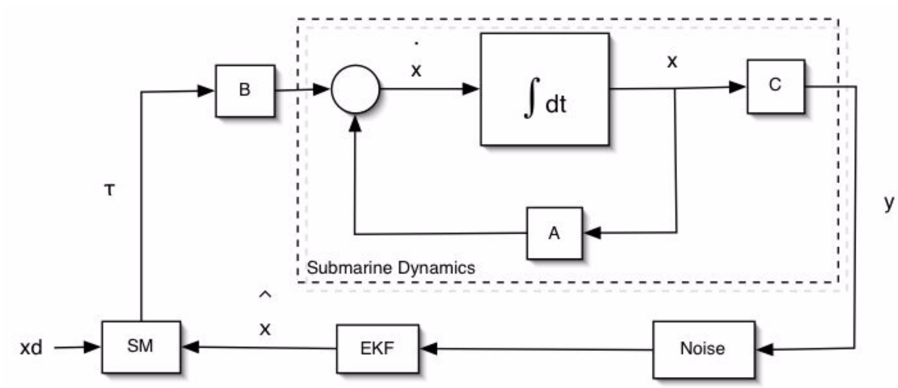

FIGURE 3. Flow chart of the simulation, showing the system dynamics, extended Kalman filter (EKF), and the sliding mode controller (SM).

IV. Results

The result presented here were obtained via a simulation conducted in Matlab. A flow chart of the simulation is shown in Figure 3.

A. Subjugator Physical Constants

The exact values of the physical parameters were not all available, thus general assumptions about the system were made. The submarine's mass and dimensions (LxWxH) were: 150 lbs and 53.37 x 29.1 x 27.45 in. The radius of gyration for x, y, and z are 24.6402, 9.7861, and 23.9856 in.

B. Control and Estimation

The submarine was commanded to follow a simple path 4 m underwater. It was a circular path in the x-y plane with a 12 m diameter and centered on the origin. The submarine is originally located at the origin which is located at the surface of the water. The position and velocity of the simulation's first 25 seconds are shown in Figure 4 and Figure 5. The sliding mode controller reaches the desired path for both position and velocity after 11 seconds. The control efforts required to achieve this are shown in Figure 6. Note that the motors are limited to their maximum output of 107 N (24 lbf) each. Notice that the control efforts contain no unwanted chattering which is the result of using the smooth *sat(s)* function instead of the *sgn(s)* function.

Finally looking at the EKF error, which is the difference between the true position and the estimated position in the x-y plane, shown in Figure 7. This plot shows the error after 300 sec., or three complete times around the desired circle. The initial line that starts at the origin and cuts through the circle was caused by the transient state of the kalman gain and error covariance matrix. Eventually both matrices settle to steady state values which occurred when the errors began to move around in the circular path shown.

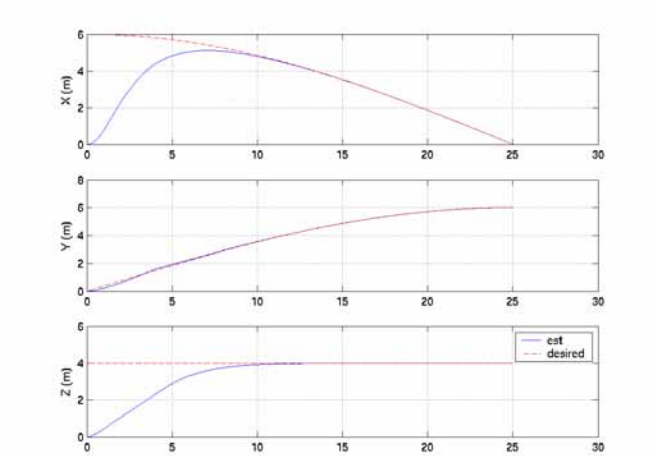

FIGURE 4. Position of the submarine while tracking a path.

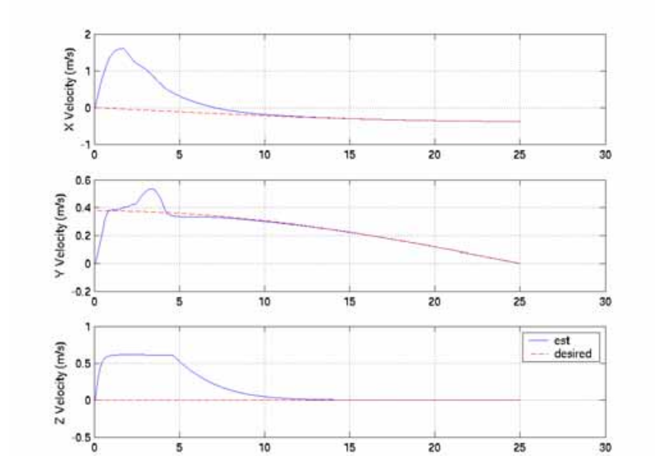

FIGURE 5. Velocity of the submarine while tracking a path.

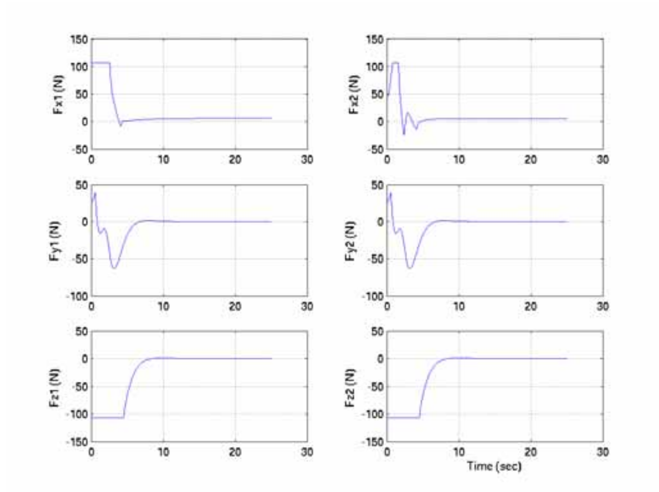

FIGURE 6. Control effort for each of the six motors which are limited to 107 N (or 24 lbf).

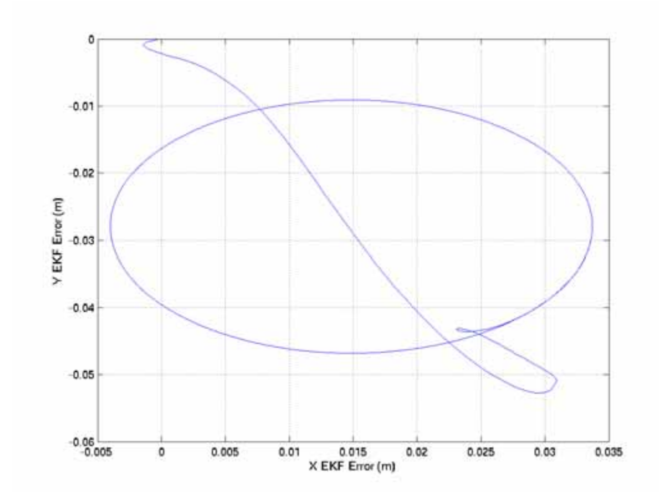

FIGURE 7. Error between the real and EKF estimated position.

C. Effects of Hydrodynamic Terms

Figure 8 shows the effects of scaling the hydrodynamics up or down on error in the system. Here the two calculations for error are the integration of absolute error (IAE) and the integration of time with absolute error (ITAE). The error was positional error only. The controller was optimized by hand to achieve the best ITAE which weights error by the time it occurs in the simulation. Thus error that occurs early, due to incorrect initial conditions, is weighted little and error that occurs later, due to tracking problems, is weighted more.

But what do the error measurements really mean as far as performance. Looking at Figure 9, which shows a top down look of the x-y plane, the effects of the scaling factor can be more intuitively understood. Remember the submarine starts off at the origin and goes to the path which lies in a circle of diameter 12 m.

The scaling factor of 0.6 produced a large amount of overshoot, while the larger scaling factor of 4 took longer to reach the desired path. The reason there was a large amount of overshoot in the 0.6 scaling factor, was the controller calculated too much control effort to reach the desired state since it was over estimating what the true hydrodynamic forces were. The results for scaling factors between 0.6 and 4 fall in between the responses shown. The response for scale factor of 1 is also shown for reference.

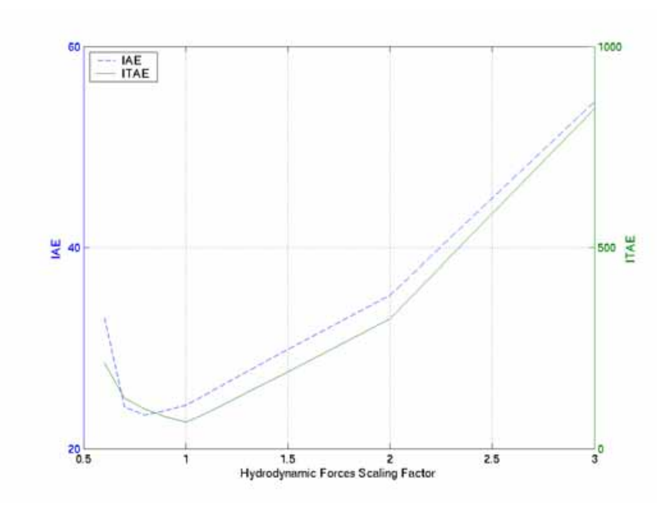

FIGURE 8. The ITAE and IAE errors for various scaling factors of the hydrodynamic forces.

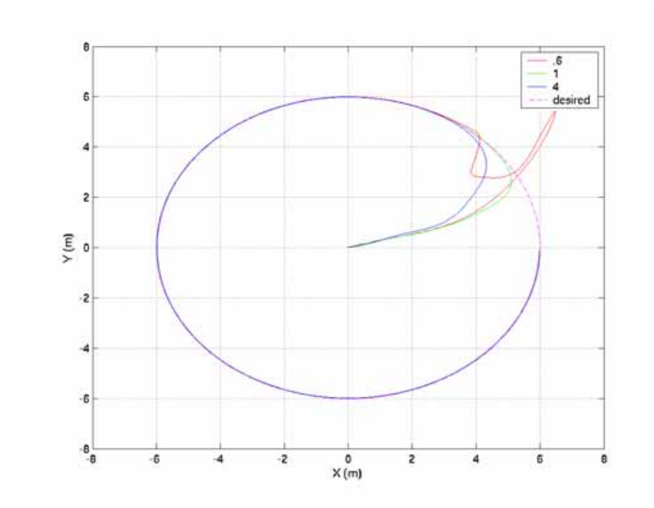

FIGURE 9. Submarine's position for various hydrodynamic scaling factors.

V. Conclusions

Sliding mode control is a very powerful control scheme for nonlinear systems with uncertainties in modelling. The results shown here tried to highlight this by showing large inaccuracies in the estimate of the hydrodynamic forces still resulted in a stable controller that followed the desired path. Although performance for the controller was best when the hydrodynamic effort were completely known, for either under or over estimation of the dynamics by as much as half still produced acceptable results.

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